

Gamma-rays from Galactic Black Hole Candidates with Stochastic Particle Acceleration

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ABSTRACT

We consider stochastic particle acceleration in plasmas around stellar mass black holes to explain the emissions above 1 MeV from Galactic black hole candidates. We show that for certain parameter regimes, electrons can overcome Coulomb losses and be accelerated beyond the thermal distribution to form a new population, whose distribution is broad and usually not a power law; the peak energy of the distribution is determined by the balance between acceleration and cooling, with particles piling up around it. Radiation by inverse Compton scattering off the thermal (from background) and non-thermal (produced by acceleration) particles can in principle explain the hard X-ray to gamma-ray emissions from black hole candidates. We present model fits of Cyg X-1 and GRO J0422 in 50 keV – 5 MeV region observed with OSSE and COMPTEL.

Subject headings: acceleration of particles — black hole physics — radiation mechanisms: non-thermal — thermal — gamma rays: observations — theory

1. INTRODUCTION

Most of Galactic black hole candidates (GBHCs) show X-ray spectra well fit by a thermal Comptonization model with temperatures $\sim 50 - 100$ keV and Thomson depths of a few (e.g., Harmon et al. 1994; Liang 1993). OSSE and COMPTEL experiments on *Compton Gamma-Ray Observatory* (CGRO), however, have recently revealed that persistent gamma-rays $> \text{MeV}$ s are being produced in some GBHCs, in particular Cyg X-1 (Johnson et al. 1993; McConnell et al. 1994a) and GRO J0422+32 (van Dijk et al. 1995). These gamma-ray tails are hard to fit with a single component Sunyaev & Titarchuk (1980) thermal model (or a more recent model by Titarchuk 1994). Hence non-thermal processes are strongly hinted by these observations.

Models of non-thermal e^\pm pairs have been studied with the emphasis on the power-law X-ray emission from active galactic nuclei (AGNs) (e.g., Fabian et al. 1986; Lightman & Zdziarski 1987; Svensson 1987; Coppi 1992), and they can be also applied to plasmas around stellar mass black holes. Those models assume that either mono-energetic or power-law leptons with a large Lorentz factor ($\gamma \gg 1$) are injected and they initiate cascade processes such as e^\pm production and Compton scattering (see Svensson 1994 for a recent review). However, those models did not specify any acceleration mechanism, thereby lacking the self-consistency in determining the particle distributions and photon spectra.

Motivated by the MeV photons from GBHCs, we study the role of stochastic particle acceleration in accreting plasmas near GBHCs. The mechanisms of wave-particle resonant interactions that lead to particle acceleration have been directly observed in solar-wind (e.g., Marsch 1991) and extensively studied in the context of solar flares (e.g., Melrose 1974; Ramaty 1979; Hamilton & Petrosian 1992; Miller & Roberts 1995). Stochastic acceleration has been applied to diffusive shock acceleration and cosmic rays (Schlickeiser 1994), the lobes of radio galaxies (e.g., Lacombe 1977; Achterberg 1979; Eilek & Henrikson 1984), and

recently, the central regions of AGNs (Dermer, Miller, & Li 1995). Here in this *Letter*, we couple the particle energization and radiative processes to determine self-consistently the steady state particle and photon distributions in GBHC environments. We then present fits to gamma-ray data of GBHCs observed with OSSE and COMPTEL.

2. THE MODEL

As a first approximation, we average all plasma and magnetic field properties over an ad hoc spherical volume around the black hole of radius R . In this homogeneous spherical emission region, we treat the relevant radiation and particle acceleration/heating processes comprehensively, but ignore details of the accretion flow, primary energy generation and viscosity, etc., by assuming naively that the total GBHC luminosity is released uniformly throughout the spherical volume. This approach allows us to first concentrate on the microphysics. We assume that the plasma cloud is in a steady state, and solve the coupled steady-state kinetic equations of particles and photons (e.g., Lightman & Zdziarski 1987). Our approach differs from previous studies in that we include the *diffusion and systematic acceleration terms* in the particle kinetic equations, both of which are derived from the stochastic acceleration. We also assume that the particles consist of a thermal and non-thermal component, coupled by Coulomb interactions.

The evolution of the non-thermal particle distribution in momentum space, $n_{\text{nt}}(p)$, can be described by the Fokker-Planck equation as

$$\frac{\partial n_{\text{nt}}}{\partial t} = \frac{\partial}{\partial p} \left[D(p) \frac{\partial n_{\text{nt}}}{\partial p} \right] - \frac{\partial}{\partial p} \left[\frac{2}{p} D(p) n_{\text{nt}} + \dot{p} n_{\text{nt}} \right] - \frac{n_{\text{nt}}}{t_{\text{esc}}(p)} + \dot{Q}_{\text{inj}}(p) - \dot{A}(p), \quad (1)$$

where $p \equiv |\vec{p}|/m_e c = \beta\gamma$, γ is particle's Lorentz factor, and $\beta = (1 - 1/\gamma^2)^{1/2}$. Here, $D(p)$ is the particle diffusion coefficient in momentum space, t_{esc} is the energy-dependent escape timescale, \dot{p} is the cooling losses (both inverse Compton scattering and Coulomb

losses are included), $\dot{Q}_{\text{inj}}(p)$ is the injection including sources from both the thermal background plasma and e^\pm pairs production, and $\dot{A}(p)$ represents the removal of leptons due to pair annihilation ($e^+ + e^- \rightarrow \gamma + \gamma$). The non-thermal particle density is given by $\int n_{\text{nt}}(p)dp$. A direct consequence of including $D(p)$ (also the difference from previous study) is the development of a population of non-thermal leptons which is accelerated out of the background thermal plasma. However, below a certain momentum p_{thr} , which we assume to correspond to the plasma temperature T_e and $\gamma_{\text{thr}} = (p_{\text{thr}}^2 + 1)^{1/2} = 1 + 4\Theta$ (see e.g., Zdziarski et al. 1990), the particle distribution should be Maxwellian, possibly due to either waves are strongly damped at those energies or “thermalization” timescale (such as Coulomb collisions) becomes the shortest. Thus all the particles with $p < p_{\text{thr}}$ are grouped into the thermal population. We concentrate on the steady state solutions ($\partial/\partial t = 0$).

We admit that the actual distribution of the turbulence are subject to great uncertainty. We assume that turbulent plasma waves are generated uniformly in the cloud and that there is a power-law distribution of turbulence energy density in wave number $W(k) \propto k^{-q}$, where $k = |\vec{k}|$, with k extending from k_{min} (which is taken to be $\sim 1/R$) to the value near electron cyclotron resonance, so that it covers both the Alfvén and whistler branches. Both $D(p)$ and t_{esc} can be evaluated from the conditions of accreting plasma (see recent treatment in Dermer, Miller, & Li, 1995). Here we combine the results from both the electron/whistler and electron/Alfvén interactions, and obtain the pitch-angle averaged momentum diffusion coefficient as

$$D(p) = \frac{\pi}{4} (q - 1) \frac{1}{t_{\text{dyn}}} \frac{\beta_A^2}{\beta} \zeta_{\text{wave}} \left(\frac{r_L}{R} \right)^{q-2} I_a(k_0, k_1) p^q, \quad (2)$$

where

$$I_a(k_0, k_1) = \frac{\beta_A^2 k_0^2 - 1}{q} \left[\left(\frac{k_1}{k_0} \right)^{-q} - 1 \right] + \frac{1}{q + 2} \left[\left(\frac{k_1}{k_0} \right)^{-q-2} - 1 \right] + \frac{\beta_A^2 k_0^2}{2 - q} \left[\left(\frac{k_1}{k_0} \right)^{2-q} - 1 \right]$$

with $t_{\text{dyn}} = R/c$, $\zeta_{\text{wave}} = U_w/U_B$ (the ratio of wave to total magnetic energy density), $r_L = m_e c^2/eB$ (the electron gyroradius), $k_1 = (m_p/m_e)^{1/2}/\beta_A$, $k_0 = (m_p/m_e)/p$, and

$c\beta_A = B/(4\pi n_p m_p)^{1/2}$ being the Alfvén velocity, where n_p is the proton number density. Here B is obtained by assuming equipartition between thermal gas pressure and magnetic pressure, and in most cases we study here, $\beta_A \sim 0.01$.

The importance of stochastic acceleration can be illustrated in Figure 1, where we plot the various timescales (in units of $t_{\text{dyn}} = R/c$) as functions of the dimensionless particle momentum p for a test electron passing through a plasma cloud with size $R \approx 1.5 \times 10^8$ cm, temperature $\Theta \equiv kT_e/(m_e c^2) \approx 0.2$, and varying density n_p . Here we assume that magnetic energy density U_B is in equipartition with thermal gas pressure ($n_p \Theta$) and $\zeta_{\text{wave}} = 0.1$. Also, $u_{\text{ph}} \equiv U_{\text{ph}}/U_B$, where U_{ph} is the soft photon energy density. All timescales are evaluated from energy change rates. The acceleration rate is calculated from $\langle d\gamma/dt \rangle_{\text{acc}} = \frac{1}{p^2} \frac{\partial}{\partial p} [p^2 \beta D(p)]$ and equation (2). The Compton+synchrotron cooling rate is $|\langle d\gamma/dt \rangle_{\text{c+s}}| = \frac{4}{3} \frac{1}{t_{\text{dyn}}} (1 + u_{\text{ph}}) \tau_p \Theta p^2$, where $\tau_p = \sigma_T n_p R$ and σ_T is the Thomson cross section. The cooling rate of lepton-lepton Coulomb interactions is taken from Dermer & Liang (1989). The diffusive escape timescale is typically the longest among all timescales and is not shown here. It is evident that, for certain parameter regimes, acceleration is more efficient than the cooling processes so that it must be taken into account in determining the particle distributions of emitting plasma.

The energy flow rates for various components in the system are outlined as follows: (A) The total gravitational energy released by accretion is characterized by the compactness ℓ , which is given by $\ell \equiv 4\pi\sigma_T R^2 \dot{u}/(3m_e c^3)$, where \dot{u} is the energy input rate per unit volume; (B) A fraction of gravitational energy, $\ell_{\text{nt}} = \epsilon_{\text{nt}} \ell$, is used to stochastically accelerate some leptons through wave-particle resonant interactions; (C) The rest of the input energy, $\ell_{\text{th}} = (1 - \epsilon_{\text{nt}}) \ell$, is used to heat up thermal leptons; (D) Soft photons, with a blackbody spectrum of temperature T_s , are injected in the cloud uniformly with the compactness ℓ_s (the source of soft photons can be optically thick accretion disks).

In determining the particle distribution and emission spectra, we have included most of the relevant microphysical processes, all of which are put into a computer code (Li 1995; see also Kusunose & Mineshige 1995 and Lightman & Zdziarski 1987). These include (A) *Non-thermal processes*: Compton cooling, Coulomb collisions and annihilation between non-thermal and thermal leptons, the flow of non-thermal pairs becoming thermal pairs due to cooling, and the escape; (B) *Thermal Processes*: Coulomb collisions between protons and thermal e^\pm , thermal bremsstrahlung and Compton scattering by thermal e^\pm , and annihilation emission by thermal e^\pm ; Thermal Compton scattering is calculated using the Fokker-Planck equation (Kompaneets equation) with the diffusion coefficient given by Prasad et al. (1988) for isotropic radiation field. When $\tau_e = \sigma_T n_e R < 2$ and $T_e > 100$ keV, however, we use a diffusion coefficient for the collimated radiation field, based on the argument by Titarchuk (1994). (C) *Radiation Processes*: Additional processes include the injection of soft photons ℓ_s , absorption due to $\gamma + \gamma$ interactions, and the escape from the system; (D) *Pair Balance*: Thermal pairs are assumed in pair balance, i.e., the pair production rate is equal to the annihilation rate for $p < p_{\text{thr}}$, and escape is neglected.

To summarize our model, for given ℓ , proton density n_p , and R , distributions of steady state particles and photons for a plasma cloud with the processes described above are obtained for given parameters such as ϵ_{nt} , ℓ_s , T_s , and q . The temperature of the plasma T_e is mainly determined by the balance among external heating (by protons) ℓ_{th} , Coulomb heating by non-thermal leptons, and cooling by thermal Compton scattering. The Thomson scattering depth for thermal leptons $\tau_e = \sigma_T n_e R$ (usually different from $\tau_p = \sigma_T n_p R$ due to pairs) is also obtained. The level of the turbulent plasma waves ζ_{wave} is unambiguously determined from the relation $\ell_{\text{nt}} = (4\pi\sigma_T R^2/3c) \int dp n_{\text{nt}}(p) \langle d\gamma/dt \rangle_{\text{acc}}$, where $\langle d\gamma/dt \rangle_{\text{acc}}$ is proportional to ζ_{wave} and $n_{\text{nt}}(p)$ is obtained from equation (1). The advantage of this approach is that we do not need to artificially assume the injection of relativistic leptons as was done previously, because the content of non-thermal particles is now self-consistently

determined from the stochastic particle acceleration.

3. RESULTS

Figure 2 summarizes our main results. Photon spectra of Cyg X-1 and GRO J0422 from OSSE (cross, Philips et al. 1995, Kroeger et al. 1995) and COMPTEL (filled square, McConnell et al. 1994b, van Dijk et al. 1995) are shown in the left panels.

(1) Cyg X-1. Our best-fit model spectrum (*left*) to Cyg X-1 and the corresponding particle distribution (*right*) are shown as the solid curves in the upper panel of Figure 2. Other parameters are given in Table 1. Clearly a deviation from Maxwellian of the particle distribution occurs at high energy tail. This occurs because the “upward” motion (in momentum space) by acceleration is balanced with the “downward” motion by cooling at a specific value of momentum, at which particles tend to “pile up”, forming a “bump” in the particle distribution (see also Schlickeiser 1984). It is from this “bump” component that most of the emissions above 1 MeV are produced via inverse Compton scattering.

Compared with the thermal models, only ℓ_{nt} and q are the new additional parameters. While the dependence of the results on q is rather weak, we demonstrate the effects of varying ℓ_{nt} in the upper panels of Figure 2 (dotted and dashed curves). Dotted curve ($\ell_{\text{nt}}/\ell = 10^{-8}$) indicates that photon spectrum from Cyg X-1 can not be fitted by pure thermal emissions. As ℓ_{nt} increases, more pairs are produced (dashed curve in particle distributions), so is $> \text{MeV}$ emission in this case (but see below). Note that the pair fraction $(1 - \tau_p/\tau_e)$ in this case is small ($\sim 3.4\%$).

(2) GRO J0422. A good fitting to GRO J0422 is shown by the solid curves in lower panel of Figure 2. For $M = 10M_{\odot}$, we find that it only needs $\ell_{\text{nt}}/\ell = 0.06$, and the particle distribution is dominated by the thermal component with only a slight deviation from the

Maxwellian distribution at higher energy due to acceleration. This is also evident from the fact that MeV emission from J0422 is much weaker, compared to Cyg X-1. We have also tried to fit J0422 by choosing a different mass but keeping other parameters to be the same as for Cyg X-1 (as suggested by the referee). Dashed and dotted curves in the lower panel show other fittings (though not as good as the $M = 10M_{\odot}$ case) to J0422 with $M = 6M_{\odot}, \ell = 15$ and $M = 4M_{\odot}, \ell = 22.5$ (see Table 1). Note that the source luminosity is $\propto \ell M$ for fixed R so it is the same for all trials. The particle distributions are now essentially thermal. Again, other parameters are given in Table 1.

Figure 2 also indicates an important feature of our model, i.e. higher ℓ_{nt}/ℓ does not necessarily result in higher flux of gamma-ray emissions. In the case of J0422, the high compactness (smaller mass) causes the cooling becoming increasingly important at high energies, destroying the otherwise suprathermal population and merging them into thermal bath. Notice that the cutoff energy in photon spectra decreases and the number of pairs increases dramatically ($\sim 26\%$ for $4M_{\odot}$) as mass decreases.

4. CONCLUSIONS AND DISCUSSIONS

We have examined stochastic particle acceleration via wave-particle resonant interactions near Galactic black holes. We find that under certain conditions, stochastic electron acceleration can overcome both Coulomb and Compton losses, resulting in a suprathermal population. Preliminary model spectra show good fits to the recent OSSE and COMPTEL observations of Cyg X-1 and GRO J0422. We find that Cyg X-1 has a much higher component in gamma-rays than J0422, as it is also evident in the data. This can be understood in terms of the higher ℓ_{nt}/ℓ and lower compactness of Cyg X-1, compared to J0422, if they have the same black hole mass. On the other hand, the high energy emissions from J0422 can be fitted by different masses of the putative black holes. But these fittings

are different from the standard thermal (i.e. e-p plasma only) models because a large amount of pairs are produced and thermalized due to the high compactness. Since both sources are highly variable, especially in gamma-ray emissions, we expect the non-thermal energy content ℓ_{nt}/ℓ might vary but the detailed microphysics of how the system partitions the energy flows is presently unclear.

The non-thermal population we obtain here is relatively soft with few pairs, not capable of explaining either the transient MeV bump discovered by Ling et al. (1987) or the COMPTEL power-law tail if it extends to much higher energies. This softness is due to our assumptions of a copious soft photon source permeating the entire emission region and *the coexistence of thermal and non-thermal particles in a single homogeneous volume*. Coulomb interaction *efficiently* transfers the non-thermal energy to the thermal reservoir which strongly limits the existence and energy content of non-thermal particle population. To achieve much harder non-thermal populations and emissions we need to postulate the existence of physically distinct regions for the thermal and non-thermal components, either in the form of a thermal disk plus non-thermal corona or jets, or thermal outer disk and non-thermal inner torus. We plan to undertake such ventures in future work. But it is highly encouraging that a simple-minded single-component spherical model with few parameters is already capable of explaining from first principles the quiescent (low-hard state) emissions of Cyg X-1 and GRO J0422 detected by CGRO.

We note, of course, that there are many other alternative interpretations of the hard tail, including the pion decay model (Jourdain & Roques 1994) and pair-dominated hot cloud model invoking quenching of the soft photon source (Liang & Dermer 1988). It is possible that particles accelerated by shocks may also account for the $> \text{MeV}$ emissions.

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Fig. 1.— Timescales of acceleration (*solid*), Coulomb loss (*dotted*), and Compton loss (*dashed*) as functions of particle’s dimensionless momentum $p \equiv \gamma\beta$. The scattering depth and soft photon energy density, indicated respectively by τ_p and u_{ph} (see text for definitions) for a given size R , are varied as $(\tau_p, u_{\text{ph}}) = (0.05, 1)$, $(0.5, 1)$, $(0.5, 10)$, and $(2, 0.1)$, for plots (a), (b), (c) and (d), respectively. Sufficient acceleration occurs in plots (a) and (b), but Compton/syn. cooling and Coulomb cooling prevent acceleration in (c) and (d), respectively.

Fig. 2.— *Upper panels:* Model fit of the emission spectrum of Cyg X-1 (*left*) and the corresponding particle distribution as function of dimensionless momentum p (*right*). Data are from OSSE and COMPTEL (*squares*) experiments. Solid curve ($\ell_{\text{nt}}/\ell = 0.15$) fits the data. Dotted and dashed curves are for $\ell_{\text{nt}}/\ell = 10^{-8}$ and 0.5, respectively. They depict the effects of changing ϵ_{nt} *only*, with other parameters fixed. Particle distributions for higher ℓ_{nt}/ℓ (solid and dashed curves) show deviations from Maxwellian distributions in the high energy tail due to acceleration. *Lower panels:* Same as the upper panels but for GRO J0422. Again, data are from OSSE and COMPTEL. The solid, dashed and dotted curves are for $M/M_{\odot} = 10$, 6, and 4, respectively. Solid curve fits the data best with $\ell_{\text{nt}}/\ell = 0.06$, and the corresponding particle distribution is very close to Maxwellian. Particle distributions for $M/M_{\odot} = 6$, 4 are essentially thermal and a large amount of pairs are produced (dashed and dotted curves).

Table 1. Model Parameters and Results ^a

Object	dist.	mass	ℓ	ℓ/ℓ_{edd}	ℓ_{nt}/ℓ	ℓ_s/ℓ	T_s	τ_p	q	R	T_e	τ_e	ζ_{wave}
Cyg X-1 ^b	2.5	10	4.5	0.02	0.15	0.07	0.1	0.7	5/3	80	139	0.725	0.059
J0422 ^c	2.5	10	9.0	0.03	0.06	0.08	0.1	0.7	5/3	80	134	0.741	0.065
J0422 ^d	2.5	6	15	0.05	0.15	0.07	0.1	0.7	5/3	80	119	0.869	0.08
J0422 ^e	2.5	4	22.5	0.08	0.15	0.07	0.1	0.7	5/3	80	111	0.944	0.07

^adistance in kpc, mass in M_{\odot} , temperature in keV, R in units of GM/c^2

^bparameters for the solid curve for Cyg X-1 in Figure 2

^{c,d,e}parameters for the solid, dashed and dotted curves for J0422 in Figure 2, respectively



